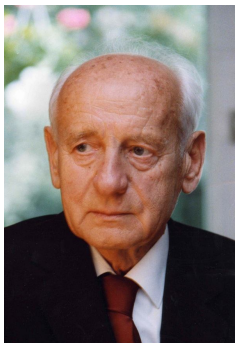
ERICH KÄHLER (1906-2000)

Born January 16th 1906 in Leipzig

Died May 31st 2000 in Wedel next to Hamburg



Biographical note

Life and works

Erich Kähler was son of a telegraph inspector and studied in Leipzig. He began to learn Mathematics at the University of Leipzig in 1924, but he also followed courses on Astronomy and physics. Kähler obtained the doctorate in 1928 with the dissertation *Über die Existenz von Gleichgewichtsfiguren, die sich aus gewissen Lösungen des n-Körperproblems ableiten* (*On the existence of equilibrium figures that are derived from certain solutions of the n-body problem*) whose advisor was Leon Lichtenstein. In 1929 he became assistant at the University of Königsberg. After meeting Emile Artin in Hamburg in 1930, he was offered a position of assistant to Blaschke, which he took in the same year. He became Privatdozent in 1930 after the submission in Hamburg of his habilitation thesis *Über die Integrale algebraischer Differentialgleichungen* (*On the integrals of differential equations*). In the academic year 1931-32, he went to Rome with a Rockefeller grant to study with the Italian geometers Guido Castelnuovo, Francesco Severi, Federigo Enriques and Beniamino Segre. In Rome he also met André Weil (who would later write on Kähler maniflods) and Tullio Levi Civita. In 1932, Kähler published *Über eine bemerkenswerte Hermitesche Metrik* (*On a remarkable Hermitian metric*) where he introduced the notion of what nowadays is called *Kähler metrics*, which Kähler manifolds are based on. In 1934 Kähler published his booklet *Einführung in die Theorie der Systeme von Differentialgleichungen* (*Introduction to the theory of systems of differential equations*) based on the same subject as the theory of Élie Cartan. In 1935 he arrived at the University of Königsberg, where he became ordinary professor in 1936. In 1938 he married Luise Günther and they had three children Helmuth (1939), Gisela (1942), and Reinhard (1948).

In 1939 Kähler was called up to serve in the Kriegsmarine. The World War II meant a break in his mathematical activity. Towards the end of the war, he was stationed at the Atlantic submarine base of Saint-Nazaire and was made prisoner by the Frenchs, who sent him to the prisoner camp of the Ile de Ré in the Bay of Biscay. Then he was sent to a camp at Mulsanne near Le Mans. Thanks to the mediation of Frederic Joliot-Curie and Élie Cartan, he could study mathematics, receive books and send mathematics papers during his imprisonment.

After his release in 1947, Kähler returned to Hamburg and met his family again. In 1948 he was appointed to the chair of mathematics at the University of Leipzig, which was vacant after Paul Koebe's death in 1945. The mathematical work of this Leipzig period was published in Italian as a special issue of the *Annali di Mathematica* with the title *Geometria aritmetica* in 1958. In this year, he went to the Technical University of Berlin after some political differences with the East German regime and stayed there until 1964, when he was offered the chair of Emile Artin in Hamburg, which was vacant since his death in 1962. Kähler held this position until his retirement in 1974. In this period his son Reinhard died in a rafting accident in 1966 and then his wife Luise became ill and died in 1970. In 1972 he married Charlotte Schulze, the widow of his brother, who had died during the war. After his retirement Kähler continued being active in mathematics also entering into chemistry, biology and philosophy. Examples of these attempts to bring together different branches of science and philosophy are *Also sprach Ariadne* (*Thus spoke Ariadne*, 1992), where he goes on Nietsche's *Also sprach Zarathrusta* and Leibniz's theory of monades, and *Raum-Zeit-Individum* (*Space-time-individual*, 1992) as a reformulation of Hermann Weyl's *Raum, Zeit, Materie*.

Kähler was elected honorary member of the Hamburg Mathematical Society in 1976, and became member of the Academy of Sciences of Saxony (1949), the Berlin Academy of Science (1955), the German Academy of Scientists Leopoldina (1957), the Accademia dei Lincei (1957), and the Accademia di Scienze e Lettere Milano (1992).

Kähler calculus

Not less than three of his contributions to mathematics bear his name: Cartan-Kähler theory of exterior systems (mainly a Cartan theory which he enriched with the Cartan-Kähler theorem), Kähler manifolds and the Kähler calculus. This calculus is to Clifford algebra of differential forms what the Cartan calculus is to exterior algebra, also of differential forms. It is contained in the following three papers:

1) «Innerer and äusserer Differentialkalkül» (*Inner and exterior differential calculus*), *Abhandlungen der Deutschen Akademie der Wissenschaften zu Berlin*, Klasse für Mathematik, Physik und Technik., **4** (1960) pp. 1-32.

2) «Die Dirac-Gleichung» (*The Dirac equation*) , *Abhandlungen der Deutschen Akademie der Wissenschaften zu Berlin*, Klasse für Mathematik, Physik und Technik., **1** (1961) pp. 1-38.

3) «Der innere Differentialkalkül» (*The inner differential calculus*), *Rendiconti di Matematica e delle sue Applicazioni* **21** (1962) pp. 425-523.

The third paper is the more comprehensive of the three. One does not need to read the first two except for the following two absences:

(a) The 1960 paper contains a half a page derivation of the formula for the Lie derivative of scalar-valued differential forms as partial differentiation with respect to one of the coordinates of an appropriately defined coordinate system. He then goes on to develop a beautiful theory of angular momentum. Unfortunately, that derivation is absent in the 1962 paper, which takes off from the result of the derivation.

(b) The first few pages of the 1961 paper constitute a very clear exposition of the form of solutions with symmetry of exterior systems. They are crucial for understanding the relationship and difference between the Dirac and Kähler equations. In a display of modesty, he refers to his equation -which supersedes Dirac's equation- with the name of the former. The contents of those pages is present in 1962 but only in an incomplete and diffuse way.

We wish to add that the knowledge of exterior calculus very much helps in the process of learning Kähler's calculus, specially if one has some knowledge of differential geometry with differential forms. Otherwise the reading of the first 22 pages of the paper would be too strenuous. If, in addition, one knows Clifford algebra, one can dispense with his ad hoc definition of the Kähler (sum of exterior and interior) derivative, which lacks transparency. The exterior derivative certainly does not imply the interior derivative, thus not their sum. But, among the infinite number of them, one stands out, since it is canonically determined by the structure of the manifold. Kähler later proceeds to show that the interior or inner derivative coincides with the co-derivative when the differential form is scalar-valued, .

There are several ways in which Kähler might have made a better use of Cartan's teachings. In the Summer School we shall present his calculus with a lesser use of components, a greater use of bases, without restriction to the Levi-Civita connection and with Clifford rather than tensor valuedness.

Kähler's calculus immediately leads very deeply into a concomitant (and ab-initio relativistic) quantum mechanics, with a more Schrödinger than Dirac flavor. It becomes more Dirac-like as one considers solutions with symmetry. His papers are full of jewels on angular momentum –and spin in particular –, charge and antiparticles, Laplacians, etc.

References

[1] Rolf Berndt, Arno Bohm, «Life of Erich Kähler» in R. Berndt and O. Riemenschneider (eds.), *Erich Kähler, Mathematische Werke, Mathematical works* (de Gruyter, Berlin, 2003) p. 3.

[2] John J. O'Connor and Edmund F. Robertson (School of Mathematics and Statistics, Univ. of St. Andrews, Scotland), *MacTutor History of Mathematics archive*: *Kähler biography*: http://www-history.mcs.st-and.ac.uk/Biographies/Kahler.html

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